Reliability-Based Damage Tolerance Methodology for Rotorcraft Structures

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ABSTRACT

This paper presents a reliability-based damage tolerance (RBDT) methodology that employs a systematic approach for probabilistic fracture-mechanics damage tolerance analysis subject to uncertainties in initial flaw, probability of detection (POD) and other random variables. By integrating a basic probabilistic analysis engine, a finite element analysis code, and a fatigue crack analysis code, the RBDT methodology has been implemented in a prototype software and used to assess the applicability and benefits of applying RBDT methodology to supplement the safe-life and deterministic-based damage tolerance design methods.

A new efficient and accurate probabilistic method, built on a two-stage conditional importance sampling approach, is presented. The first stage computes risk, without inspection, using the most probable point (MPP)-based importance-sampling technique combined with a new error-checking method. The second stage computes risk, with inspection, by simulating inspection and maintenance effects using the samples generated from the Stage 1 failure domain. The error-checking procedure addresses the major shortcoming in the MPP-based approximation methods and leads to a robust and efficient sampling method.

For inspection optimization and maintenance planning, this paper proposes a strategy to significantly speedup the optimization process by re-using the crack growth histories for risk and risk reduction computations without additional stress and life analyses.

The results from several demonstration examples suggest that the improved two-stage importance-sampling method is well suited for RBDT analysis, particularly with inspection planning.

INTRODUCTION

Currently most computational fracture mechanics methods and tools used in the design of rotorcraft structures rely on deterministic analysis. In reality, many design parameters, including defect or flaw characteristics, crack growth law, crack detection, loads, and usages are statistical in nature. To account for the uncertainty in a deterministic-design framework, conservative assumptions based on the safety-factor approach and the worst-case scenarios are often employed to help ensure structural reliability and safety. However, in the deterministic framework, the reliability and safety cannot be quantified. In addition, it is fair to point out that the conservatism of a deterministic approach can result in suboptimal design by (1) forcing the application of an unnecessary certification maintenance requirement or (2) forcing the designer to use overly conservative design philosophies instead of the preferable more reliable and robust damage tolerance philosophy.

A comparison between deterministic and probabilistic damage tolerance analyses is shown in Table 1. As shown, the deterministic approach applies, either explicitly or implicitly, the safety factor or bounding approaches in several key design variables. As a result of the safety factors and bounds, reliability can, in fact, be designed in, but the degree of reliability cannot be

quantified. On the other hand, the probabilistic approach requires relatively more precise characterizations of the input uncertainties based on data and expert knowledge. This approach can provide quantitative risk information to complement deterministic design methodology and has the potential to reduce unnecessary conservatism.

Table 1. Deterministic vs. Probabilistic Damage Tolerance

	Deterministic	Probabilistic
Principles	Bounds, Safety Factors	Probability & confidence
Flaw/defect size	A given crack size	Distribution of crack size
Existence	Probability = 1	0 <= Probability <= 1
Inspection schedule	Life/N	Max. risk reduction
Safety measure	Safety margin	Reliability = 1 – p_f
Other variables	Bounds, Safety Factors	Distributions

In this paper, the focus of the RBDT methodology development is on rotorcraft structures. The applicability of RBDT and the computational efficiency and accuracy of the two-stage conditional importance-sampling

method will be assessed based on simple but representative rotorcraft structures with realistic rotorcraft load spectra.

RELIABILITY-BASED DAMAGE TOLERANCE METHODOLOGY

RBDT ANALYSIS FRAMEWORK

Rotorcraft structures are subjected to in-service inspections and subsequent maintenance actions, if warranted, to maintain reliability and minimize risk. Since the effectiveness of an inspection depends on the POD and the location and size of a crack, the optimal design and maintenance of reliable structures requires methods to evaluate time-dependent reliability analysis subject to inspections. For practical purposes, such methods should be fast to provide a quick turnaround of reliability analysis for design and inspection planning.

Based on the recent FAA research work for risk assessment of aircraft turbine engines [Ref. 1] and the earlier NASA Probabilistic Structural Analysis Methods program [Ref. 2], a framework for rotorcraft RBDT has been developed and summarized in Figure 1.
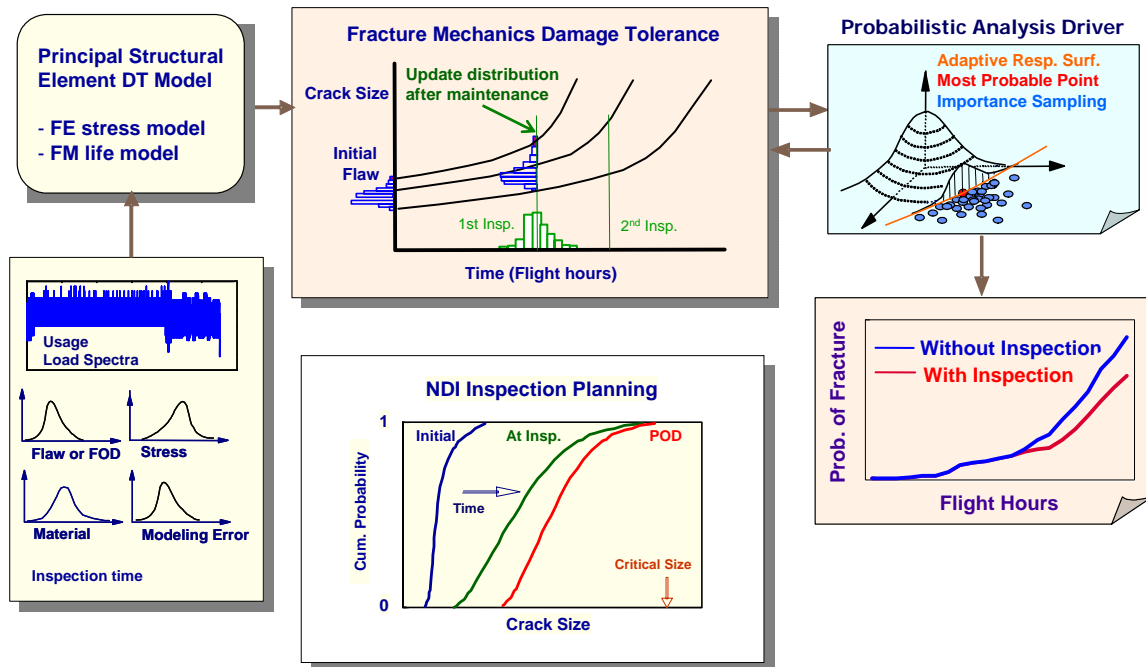

Figure 1. Rotorcraft Reliability-Based Damage Tolerance Analysis Framework

A comprehensive RBDT framework should include a wide range of uncertainties including:

- Random or uncertain parameters in material (e.g., threshold of the stress-intensity factor, modulus of elasticity)
- Defect or flaw (including size, shape, and location, and the frequency of occurrence)

- Loading, type of usage (with frequency of occurrence)
- Finite element model (including modeling error)
- Crack growth model (including modeling error)
- Maintenance (including inspection schedules, frequency of inspections, probability of detection curves, repair/replacement methods and effects)
- Human factors

The effects of human factors can be reflected in modeling errors, PODs, and other random variables by inflating the standard deviations.

PROBABILITY OF FAILURE FORMULATION

Given an initial flaw, the flaw size and stress-intensity factor K will increase with the loading cycles. A fracture failure will occur when K reaches or exceeds the fracture toughness K_c:

$$K(X_1, \cdots, X_n, N_s) \geq K_c \qquad (1)$$

The stress-intensity factor is dependent on the service life, N_s, and the random variable vector X that includes all the random variables except inspection-related parameters. An alternative failure limit state is:

$$g(X, N_s) = N_f(X) - N_S \qquad (2)$$

where N_f is the fracture mechanics life-to-failure. Without considering inspections, the probability-of-failure <u>without inspection</u>, p_f^o, expressed as an integral, is:

$$p_f^o = \text{Pr.}[N_f(X) \leq N_s] = \int \cdots \int_{N_f \leq N_S} f_X(X)dX \qquad (3)$$

in which $f_X(X)$ is the joint probability density function of the input random variables. When the limit state function involves finite element and other numerical models, computing probability of failure can be extremely computation-intensive. If inspections were considered, p_f would be even more difficult to compute because N_f would be a function of inspection time, POD, and post-inspection actions (repair or replacement). For example, for a single-inspection case, the p_f just before the inspection can be computed using Eq. 3 with N_s set at the time of inspection. Immediately after the inspection, the defect size distribution needs to be adjusted based on the POD and repair/replacement plans such that the new population of the defects is a mix of the two populations. The p_f after the inspection time needs to be computed by re-applying Eq.3 with an adjusted $f_X(X)$. The sum of the two (just before and after the inspection) integrals is the <u>cumulative</u> p_f (Ref. 3) with inspection. The difficulty in this summation approach is to compute the adjusted $f_X(X)$.

While there are many fast approximation methods (such as the first-order reliability method) for solving Eq.3, no general approximate solutions exist for computing p_f with inspection. The method proposed in Ref. 4 adjusts Eq. 3 by multiplying $f_X(X)$ by the probability of non-detection (PND), conditioned on the inspection time, i.e.,

$$p_f = \int \cdots \int_{N_f \leq N_S} f_X(X)PND(a \mid X(t))dX \qquad (4)$$

and computes Eq. 4 by an approximation procedure similar to but cruder than the FORM method. This approach implicitly assumes that a defect that would fail if no inspections, will not fail if the defect is allowed to grow to infinity and is found from any of the inspections. The approach also implies that no future failure will occur after the defect has been detected. In reality, some defects may fail early and be unavailable for future inspections, and when a defect has been detected, there is no guarantee that a repair or replacement will eliminate a future failure. Therefore, the approach is nonconservative.

Eq. 4 suggests the high level of difficulties in analytically treating general inspection and maintenance effects. An approach that includes some kinds of sampling-based simulation, such as the conditional importance-sampling method described below, seems more promising.

AN IMPROVED TWO-STAGE, CONDITIONAL IMPORTANCE-SAMPLING METHOD

The approach described here combines the MPP-based methods (e.g., FORM), the importance sampling, and a two-stage (without and with inspection) conditional probabilistic analysis process.

When a reliability problem involves inspection and replacement, the FORM approach becomes ineffective because it is computationally difficult to update the defect distribution after each inspection and maintenance. The DARWIN approach [Ref. 1] addressed the problem by using a two-stage approach, where in the first stage, the p_f without inspections is first calculated by a three-dimensional numerical integration (and avoided the FORM approximation), and in the second stage, conditioned on the Stage 1 result, importance sampling is conducted where the samples are generated using conditional distribution functions. This approach was both accurate and fast, but it was tailored for three random variables (defect size, stress scatter factor, and life scatter factor) and was intended for the turbine rotor applications. However, the numerical integration approach is not well suited for problems with a larger number of random variables due to the difficulty in high-dimension integration. In addition, the DARWIN approach assumed the stress to be a user-defined scatter factor multiplied by a single finite element (FE) stress result. There was no provision for integrating with stochastic FE-based analysis.

For rotorcraft RBDT however, more random variables are generally needed and, therefore, more general methods are required.

This study has generalized the DARWIN and the special version of FORM approach [Ref. 4] by using an error-controlled FORM procedure to deal with more random variables and with more general maintenance actions (remove, repair, or replacement with new/old parts). A key feature of the new error-controlled procedure is that it extends the service life and generates more samples for checking the FORM results, including not only the probability of failure but also the MPP. Having a robust error-control/checking procedure is critical because it has been widely recognized by the aircraft industry that the FORM method may produce large errors for some highly nonlinear or ill-behaved functions without providing warnings. Lack of an error-checking procedure was, in fact, a major reason DARWIN did not adopt the FORM approach.

To summarize, the DARWIN approach computes the probability of failure in three steps. The first step computes the probability of failure without inspections, p_f^o. The second step simulates inspection by using random samples generated only from the failure region of Step 1. Finally, the conditional probability of failure from Step 2 is multiplied by the p_f^o from the first step to reach the final probability of failure with inspection, p_f. The concept of the approach is illustrated in Figure 2, assuming that the defect size is the only random variable in the crack growth model.
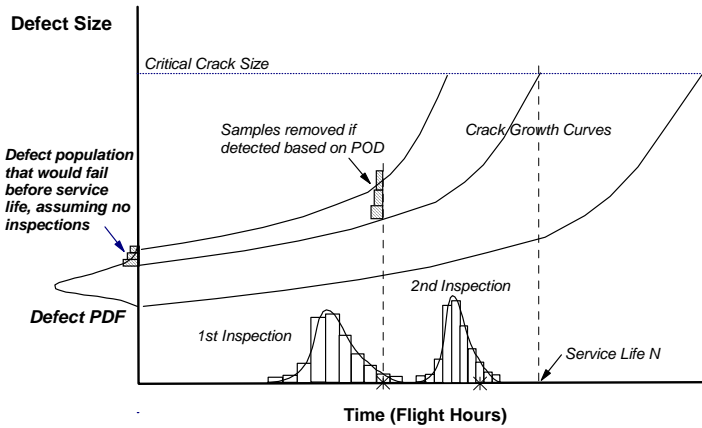

Figure 2. Concept of the Importance-Sampling-Based Simulation Method

The proposed generalized approach for more than three random variables consists of four steps. First, the inspection-free risk p_f^o is computed using the standard FORM method, i.e., compute the MPP and use the hyper-tangent surface at the MPP to estimate the probability of failure [Refs. 5-6]. The more accurate

second-order reliability methods can be used with additional computations.

In Step 2, a second FORM analysis is conducted for an adjusted, more conservative, service life. The second limit state is defined as:

$$g_A(\boldsymbol{X}, N_s) = N_f(\boldsymbol{X}) - A \cdot N_S \qquad (5)$$

where $A > 1$ is an adjustment factor selected to allow for generating more samples in Step 3 to check the solution from Step 1. The selection of A > 1 ensures the second failure region contains the first failure region. The value of A can be based on the FORM result to predict a slightly larger (say 20% larger) p_f. Therefore, the adjusted probability of failure, p_f^A, is greater than p_f^o. Because we can use the first MPP as the initial guess to search for the second MPP, the computational cost for the second FORM solution is expected to be significantly smaller than the original FORM solution.

In Step 3, a number of samples is selected to generate failure samples using the second MPP from Step 2. These samples are used for the following analyses:

(1). The samples with lives shorter than N_S are used to compute a new p_f^o using:

$$p_f^o = p_f(\text{2nd FORM}) * \frac{\text{Number of samples with lives} <= N_S}{\text{Total number of samples}} \qquad (6)$$

If the calculated number is close to the first FORM solution, it should provide some improved confidence that Eq. 6 is probably a good estimate. In addition, the first FORM MPP should be compared with the sample-based MPP to ensure that a correct MPP has been found. On the other hand, if the two numbers are significantly different, the (first) FORM solution is probably not accurate and Eq. 6 with larger A values and more samples should be used. For the cases where FORM does provide reasonably accurate solutions, Eq. 6 provides an efficient way to check and enhance the FORM solution.

(2). The samples with crack growth lives shorter than N_S are used to simulate the inspection and maintenance processes, and compute conditional probability of failure, p_f^c, i.e., the probability of failure with inspections conditioned on the population of those components with lives shorter than N_S. The simulation method is illustrated in Figure 3. For simplicity, A = 1 is used in Eq. 5. All the defects that violate the limit state are grown to failure. The histories of the crack growths should be recorded for later use.
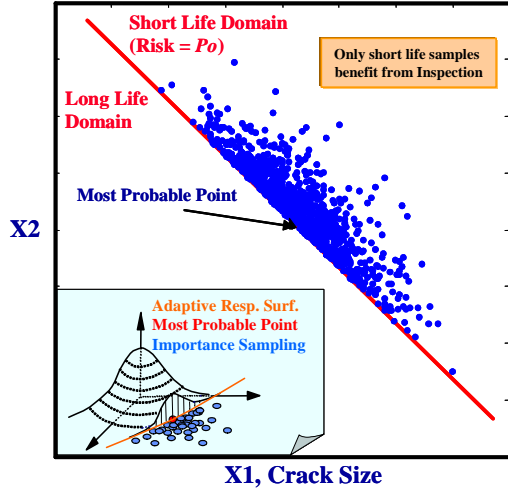
Figure 3. Compute P_f With Inspection Using Conditional Probability and Importance Sampling

For each sample, at each inspection, the POD is used to simulate the effect of inspection. When a defect has been detected, the defective component will be replaced or repaired using the corresponding defect distributions to generate a new defect size, and the crack growth process will continue until a failure occurs, the next inspection time is reached, or the service life is reached.

Finally, Step 4 computes the probability of failure with inspections as follows:

$$p_f = p_f^c \cdot p_f^o \tag{7}$$

The proposed method can be applied to many structural systems as long as the FORM solution provides a reasonable approximation such that more accurate solutions can be estimated using Eq. 6.

Step 2, proposed above, is believed to be a more robust and accurate approach. However, because of the time constraint, this approach has not been implemented in the prototype software. Instead, A = 1 in Eq. 5 has been implemented, which is sufficient for the demonstration examples.

FLAW LOCATION UNCERTAINTY AND RISK INTEGRATION

Generally, flaws are randomly distributed within a structural system. For multiple components and multiple defect locations, a system reliability method that accounts for multiple g-functions is needed. For example, if a system consists of m possible failure events, the system risk p_f is the probability union of the m failure events:

$$p_f = \Pr.[F_1 \cup F_2 \cdot \cdot F_m] \tag{8}$$

The failure events are correlated if the g-functions have common random variables such as loads. For independent or weakly correlated events, the above equation can be simplified as:

$$p_f = 1 - (1 - p_1)(1 - p_2) \cdot \cdot (1 - p_i) \cdot \cdot (1 - p_m) \tag{9}$$

For small p_i, the above equation can be simplified as:

$$p_f = \sum_{i=1}^{m} p_i \tag{10}$$

which can be written as:

$$p_f = \sum_{i=1}^{m} \alpha_i \cdot p_i^c \tag{11}$$

where α_i is the defect occurrence probability and p_i^c is the (conditional) probability of failure, given the defect. Eq. 11 allows the flaw location uncertainty to be modeled by treating a component as a summation of multiple zones where each zone potentially has a flaw with an assigned probability. An advantage of the zone-based approach is that it allows zone-dependent parameters.

EFFICIENT INSPECTIOIN TIME OPTIMIZATION

The ultimate objectives of the inspections, of course, are to detect and replace/repair defective parts before they fail. Theoretically it is possible to inspect and replace/repair as frequently as needed so that the growth of risk over time is only due to the increasing chances of extreme loads but not due to strength reduction. In practice, the number of inspections is necessarily constrained by the costs and the equipment downtime. In this paper, we will confine the discussion to a single inspection and focus on how to determine the optimal inspection time.

Assuming one inspection at $t = t_{INSP}$, the POD of the entire population is:

$$\text{Population POD} \equiv p_{POD}$$
$$= \int_0^\infty POD(a) \cdot f(a_{\text{Survived parts}}(t_{\text{Insp}}))da \tag{12}$$
$$= E[POD(a_{\text{Survived parts}})]$$

where $f(a_{\text{Survived parts}}(t_{INSP}))$ is the probability density function (PDF) of the survived defective parts at the time of the inspection. These detective parts include the parts that would not fail at $t = t_{Service}$. If we limit the survived pdf to those "critical parts", defined as those

parts that have survived past $t = t_{INSP}$ but would fail by $t = t_{Service}$, if not detected, the probability of effective detection is:

Probability of Effective Detection $\equiv p_D$

$$= \int_0^\infty POD(a_{\text{Crit. parts}}) \cdot f(a_{\text{Crit. parts}}(t_{\text{Insp}})) da \qquad (13)$$

$$= E[POD(a_{\text{Crit. parts}})]$$

The probability of the critical parts from the original population is:

Probability of Critical Parts $\equiv p_C$

$$= p_f^o(t_{\text{Service}}) - p_f^o(t_{\text{Insp}}) \qquad (14)$$

Therefore, the maximum reducible risk is the product of Eq. 13 and Eq. 14:

Reducible Risk $\equiv p_r$

$$= p_C \cdot p_D \qquad (15)$$

$$= [p_f^o(t_{\text{Service}}) - p_f^o(t_{\text{Insp}})] \cdot E[POD(a_{\text{Crit. parts}})]$$

and the risk with inspection at $t = t_{Service}$ is:

Risk With Inspection $= p_f^o(t_{\text{Service}}) - p_r \qquad (16)$

Eq. 15 is the *maximum* reducible risk because, by chance, the replaced/repaired parts can still fail by $t = t_{Service}$. However, assuming small probability of failure and independent events, the probability of the "second" failure will be significantly smaller. For the purposes of efficient approximation, this secondary probability will be ignored.

It is important to point out that Eq. 15 suggests that by saving the crack growth histories for all the importance samples without inspections, we can use the stored time-dependent defect sizes to compute risk reduction for any $t < t_{Service}$ without additional stress and life analyses. Thus, Eq. 15 is an efficient approximate formula for inspection optimization.

FURTHER COMPUTATIONAL RELIABILITY ANALYSIS ISSUES

Because of the unique loading environments in rotorcraft maneuvering relative to aircraft missions, existing reliability-based methods and codes for turbine engine and fixed-wing aircraft are not directly applicable. At present, probabilistic approaches are more widely used in designing aircraft engines and are also increasingly being used in designing other high-cost products such as aerospace and automotive systems. In selecting reliability methods, there are two important factors to consider: efficiency and accuracy. At present, the efficient probabilistic methods such as the FORM method work well for smooth, well-behaved functions.

Unfortunately, for nonsmooth or highly nonlinear functions, the MPP-based algorithms may produce large errors without providing error estimates or any warnings. On the other hand, the brute force Monte Carlo method can ensure that the error is controlled, but the analysis is time consuming and, in most FE-related applications, is impractical to use.

In general, a rotorcraft RBDT analysis requires a probabilistic finite element analysis and a probabilistic fracture mechanics analysis that can be very computationally demanding. To reduce the CPU time, the following computational strategies can be recommended to help achieve the balance of efficiency and accuracy.

RESPONSE SURFACE APPROXIMATION

Response surfaces are approximate models of the original models and are useful for approximating well-behaved functions. For example, if the stress response is dominated by the loads, a quadratic polynomial equation model may be an adequate representation of the original stress function over the range of input variations. However, in general, a checking procedure is needed to control modeling errors.

DECOUPLED STRESS AND LIFE ANALYSES

If fracture mechanics life is weakly related to the random parameters that cause the stress variation, the stress and life responses can be decoupled. This allows us to have the options to

- develop a stress response surface first and use it later for probabilistic life analysis.

- conduct a full probabilistic stress analysis to develop stress distribution and store it for probabilistic life analysis.

The second option is more accurate but usually requires more computational time. For a specific application and with enough user experience, it may be possible to use the nominal stress (based on one FE run) as the mean stress and assume a coefficient of variation to develop a lognormal or other appropriate distributions. This option requires only one FE stress analysis. However, the approximation is adequate only if the stress variation is not a major contributor to the probability of failure.

MPP-BASED METHODS WITH ERROR CHECKING

To address the lack of error control issue, one strategy is a progressive MPP method [Ref. 7], which progressively applies better MPP approximation models to check solution convergence. The approach is expected to provide a confidence indicator for the analysis result.

RBDT PROTOTYPE SOFTWARE

An ideal tool for Rotorcraft RBDT should have the following major capabilities:

- Efficient, accurate, and robust probabilistic analysis methods

- Probabilistic engine seamlessly integrated with major FE and fracture mechanics (FM) codes for various critical components

- Fully integrated and automated probabilistic life analysis with inspection

For broader applications, the ideal tool should be modularized such that the three key modules, probabilistic analysis, FE analysis, and fatigue fracture mechanics, are replaceable by user-preferred codes, provided the input and output of the codes follow certain file formats.

In this study, the ANSYS code was selected for the finite element stress analysis and the NASGRO (Version 3.0) code was selected for the fracture mechanics analysis. However, a modularized batch mode software framework has been developed to interface with other codes.

An extended research version of ProFES for the feasibility study was developed by (1) adding batch mode capability and generic function interface, (2) interfacing with FE and FM codes using text I/O files (generic function interface), (3) adding new capabilities to analyze multiple structures or one structure with multiple defect locations, (4) adding inspection simulation using importance sampling, (5) adding routines to postprocess sampling-based sensitivities, and (6) adding user-defined defect size distribution and POD.

ROTORCRAFT DEMONSTRATION EXAMPLES

Two examples are selected to demonstrate the RBDT methodology for rotorcraft applications. These are the first set of examples that are representative but simple enough for the feasibility study. Additional industry examples should be used to further test and improve the methodology and identify critical data needs.

The first example is a plate model with 190.5 hours of FELIX/28 rotorcraft load spectra, and the second is a spindle lug model with a 1-hour rotorcraft load spectra. To simplify the demonstrations, the detected defective parts are replaced by perfect parts with no defects. Note, however, the RBDT prototype software can simulate repair and replacement by using user-defined defect size distributions.

PLATE MODEL

The structure model selected to represent a rotorcraft structural part is a plate with a hole, as shown in Figure 4. A corner crack is assumed at the hole as indicated.
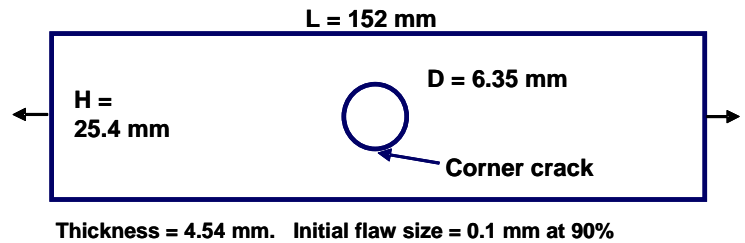
Thickness = 4.54 mm.   Initial flaw size = 0.1 mm at 90%

Figure 4. Plate Model (Ref. 8)

The selected rotorcraft load spectra is FELIX/28, as shown in Figure 5.   The spectra are based on the main rotor blade of a military helicopter with four mission types and 140 flights [Ref. 8].
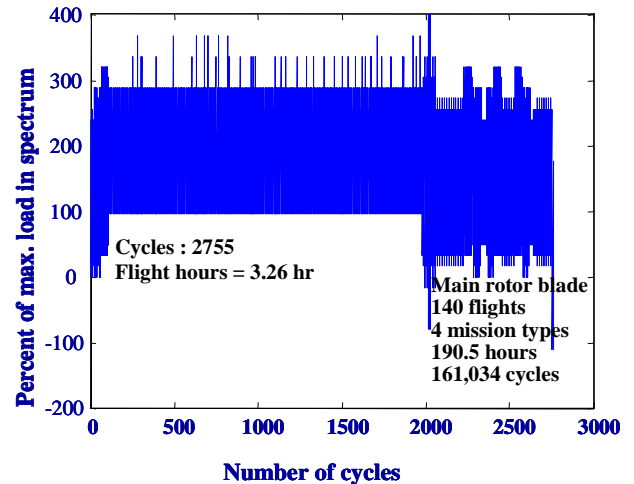
Figure 5. Felix/28 helicopter load spectra (Ref. 8)

Plate Model Using ANSYS FE Stress Model

To demonstrate a completely integrated FE-based RBDT analysis, we developed a ¼ model of the plate using the ANSYS software, even though an analytical stress solution was available. The ANSYS model, the load spectra, and the NASAGRO model are summarized in Figure 6. The random variables are defined in Table 2.   The initial crack size distribution is based on the equivalent initial flaw size (EIFS) distribution derived from the stress-life experiment, as described in Ref. 9.

This example represents a "full" stochastic stress and crack growth RBDT analysis with random variables in FE and crack growth models as well as POD. The inspection time is fixed but can easily be modeled as random. The selected mean thickness of 4.54 mm results in $p_f = 0.1$, a probability large enough to allow the analysis to be done within a reasonable time frame.
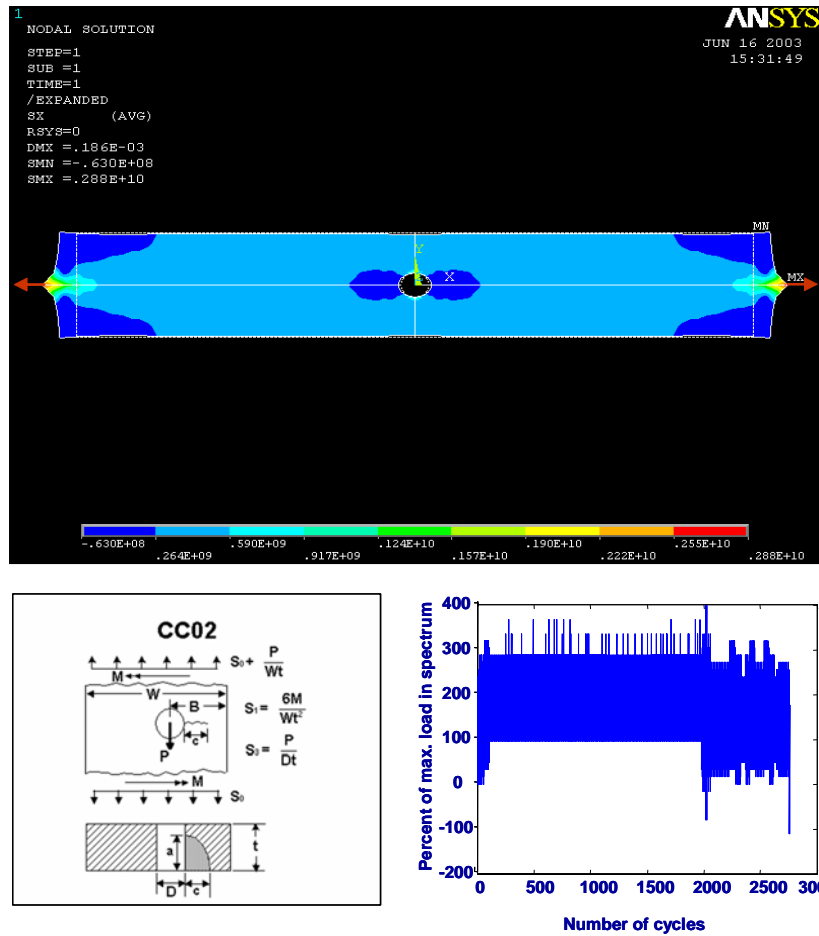
Figure 6. Plate With Hole Results Using ANSYS FE Model (Full RBDT Analysis)

Table 2. Random Variables for the Plate Model

	Distribution	Mean	Std. Dev.	COV(%)
Thickness (mm)	LN	4.54	0.023	0.51
Max. Load (N)	N	23658	2200	9.30
Initial Flaw Size (mm)	User-defined	0.074	0.0224	30.2
Delta K_{th}	LN	156.37	10	6.40
Life Scatter	LN	1	0.1	10.0

For a service life of 750 hours, the p_f result for a representative POD with and without an inspection at either 400 or 500 hours is shown in Figure 7(a). A two-parameter (median and scale) log-logistic POD model described in Ref. 10 is used. Figure 7(b) compares the POD with the cumulative distribution functions (CDF) of crack size at 400 and 500 hours. The information is useful for inspection optimization. For example, the POD will be more effective at 500 hours than at 400 hours because the defect CDF is closer to the POD.
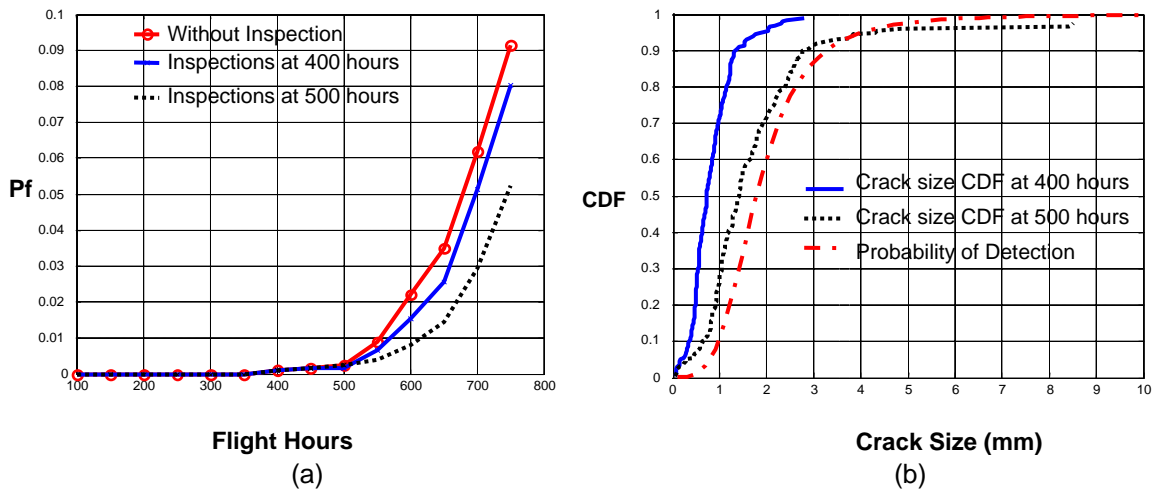

(a)

(b)

Figure 7. Plate Model Result Using ANSYS FE Model (Full RBDT Analysis)

The total number of FE and NASGRO analyses was 220. In general, the number of FE analyses is an increasing function of the number of input random variables associated with the stress, while the number of NASGRO analyses depends on the number of input random variables associated with the component life. Additionally, each importance-sampling point requires one FE analysis and one NASGRO analysis.

The total CPU time using a desktop PC for the analysis is 240 minutes. Of that, 216 minutes are for the NASGRO analyses. In general, for large complicated FE models, the required CPU time for FE may become dominant. For the NASGRO analyses, the CPU time needed is roughly proportional to the service life.

Using the samples generated from the importance-sampling method, the risk sensitivities, defined as the sensitivity of the p_f with respect to each input standard deviation, are calculated for the "with" and "without" inspection cases [Refs. 1, 11]. The sensitivities are then normalized so that they sum to one. Figure 8 compares the sampling-based sensitivities with the standard FORM-based sensitivities, also normalized. The results suggest that both methods provide consistent sensitivities. In this example, the uncertainty in the applied load is clearly the dominant random variable.
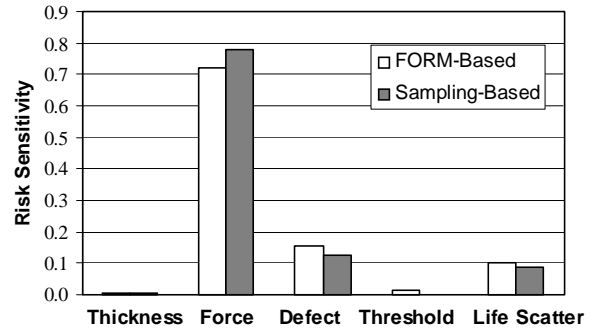

Figure 8. Risk Sensitivities for the Plate Model

Plate Model Using Simplified Random Variables

To further investigate the RBDT inspection issues, the stress is fixed and the random variables are limited to initial defect size and POD. A thickness of 4.54 mm is selected to meet the reliability target of 0.9 with a service life of 1000 hours.

Figure 9 shows the results of p_f for the three POD curves representing poor, practical, and excellent NDI capabilities. A single inspection time at 500 hours is used to compare the effects of these curves. The result shows that the POD and optimal inspection time can have significant impacts on risk management.
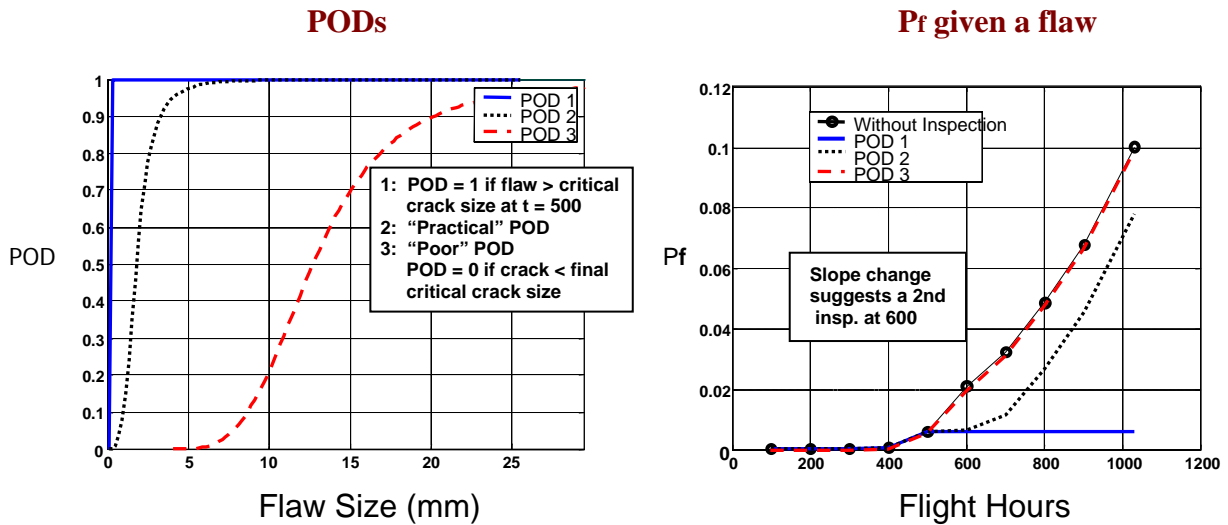

**Figure 9. Risks for the Simplified Plate Model Example for Three PODs**

LUG MODEL

The second selected model is a helicopter spindle lug shown in Figure 10 [Ref. 10]. Figure 11 shows the NASGRO model and the 1-hour load spectra used in the RBDT analysis. The random variables are listed in Table 3. A simplified stochastic life model used is defined as:

$$N = C \cdot N_{model} \qquad (17)$$

where N_{model} is the life model, and C is the life scatter random variable. The load random variable represents the point load applied to the center of the pin, P, in Figure 11. The initial defect size distribution and the POD are the same as the ones in the previous plate with a hole model.

Table 3. Random Variables for the Lug Model

	Distribution	Mean	Std. Dev.	COV(%)
Thickness, t (mm)	LN	28	0.14	0.50
Max. Load (N)	LN	145000	10000	6.9
Initial Flaw Size (mm)	User-defined	0.074	0.0224	30.2
Delta K_{th}	LN	48	4	8.33
Life Scatter	LN	1	0.1	10.0

P = 140 KN
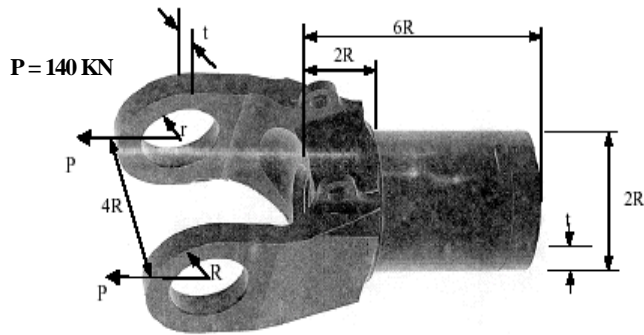
Reference R = 0.25 m, Thickness = 67 mm, Initial flaw size = 0.4 mm

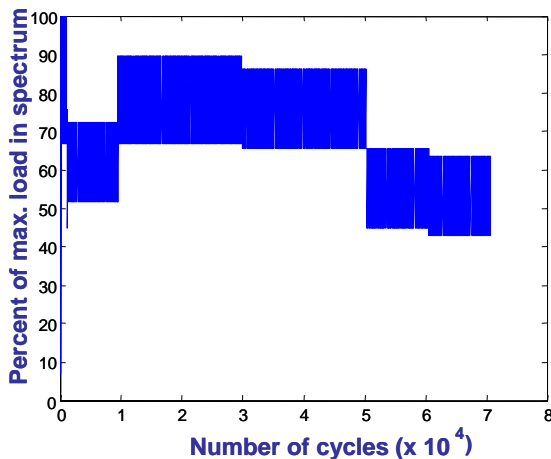**Figure 10. Spindle Lug (Ref. 10)**
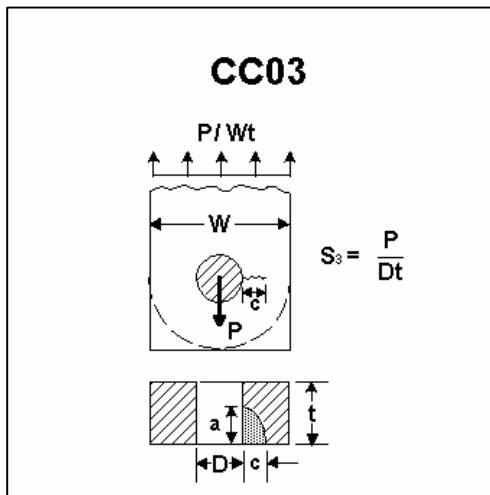
Figure 11. Spindle Lug Example

Since an analytical stress formula is available, no FE analysis is needed. Using the importance sampling method, 200 samples are used for simulating the inspection. Figure 12 shows that the inspection at 400 hours does not result in a significant risk reduction. The CPU time for the analysis is 150 minutes, mostly for NASGRO analyses. The risk sensitivities are shown in Figure 13. The most significant uncertainty is initial defect.
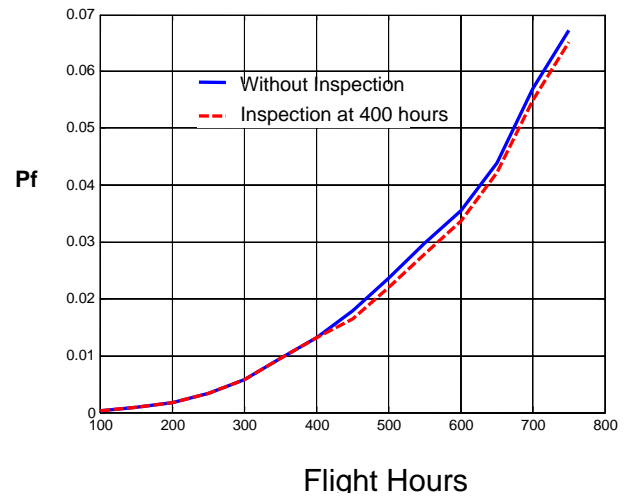

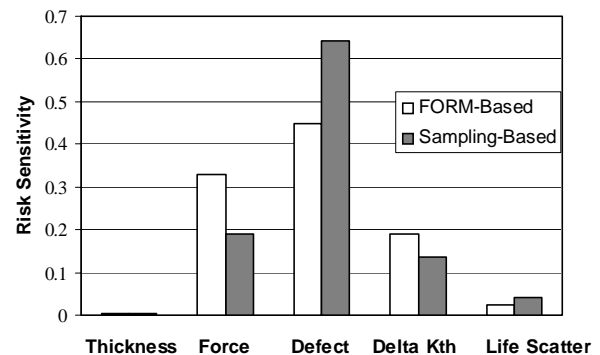**Figure 12. Risk Result for the Lug Model**

Figure 13. Risk Sensitivities for the Lug Model

CONCLUSIONS

An RBDT methodology has been developed and demonstrated using a software tool that integrates reliability methods (ProFES), a finite element analysis code (ANSYS), and a fracture mechanics code (NASGRO 3.0). The developed RBDT methodology includes an efficient two-stage conditional importance-sampling method for p_f with inspection analysis. The examples suggest that the approach is well suited for inspection planning, and seems quite generic for application to other structures, including aircraft engines and wings. Additionally, based on a sampling-based sensitivity analysis method, the generated samples can be directly used to identify and rank input random variables.

The study demonstrates that the RBDT methodology can be implemented by integrating existing software tools with added robust and efficient reliability analysis methods. The RBDT approach can provide quantified reliability, identify important variables, and support optimal inspection planning. It provides additional information to supplement the safe-life and deterministic damage tolerance approach.

From the examples, we have learned that even with the efficient sampling method, the NASGRO analyses are time consuming (several hours). The CPU time would increase even more if larger FE models were used. To reduce the time, approximate stress, crack size, and life models are needed. More advanced response surface methods with error-checking procedures are highly desirable.

Because of the time constraint, Step 2 (Eq. 6) in the importance-sampling approach that was designed to check/enhance the FORM solution has not been implemented. The proposed checking procedure is believed to be important and should be implemented in future RBDT software. Further study is highly desirable to determine how to select the adjustment factor, A, in Eq. 5, and to investigate the performance of the method.

Additional research and development is recommended to:
- use industrial models and data to further test and improve the methodology and identify critical data needs.

- further develop an efficient inspection-time optimization methodology for multiple inspections, and study the key issues related to POD and inspection/maintenance planning.

- develop automatic error-checking methods for the response surface methods and the MPP-based reliability analysis methods.

- develop a fully automated analysis software system that can handle multiple critical components and multiple locations and generate optimal inspection and maintenance plans.

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